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The cap in the hat: unoriented 2D strings and matrix(-vector) models

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Abstract: We classify the possible bosonic and type 0 unoriented open and closed string theories in two dimensions, and find their dual matrix(-vector) models. There are no RP^2 R-R tadpoles in any of the models, but many of them possess a massless tachyon tadpole. Thus all the models we find are consistent two-dimensional string vacua, but some get quantum corrections to their classical tachyon background. Where possible, we solve the tadpole cancellation condition, and find all the tachyon tadpole-free theories.

Keywords: Superstrings and Heterotic Strings, D-branes, Matrix Models.

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Contents

1. Introduction

The solution of the $c = 1$ matrix model in the double scaling limit provided an exact realization of two-dimensional string theory [1]. This has been tested to high precision in perturbation theory (for a review see [2, 3]). The matrix model also predicted a non-perturbative effect scaling as $\exp(-1/g_s)$, which was conjectured to be a generic feature of string theory [4]. This conjecture was confirmed by the discovery of D-branes in string theory [5, 6, 7]. However, lacking a clear understanding of D-branes in two-dimensional string theory, it was difficult to test the matrix model beyond perturbation theory (for some recent attempts see [8]). Furthermore, the matrix model created some problems of its own, such as the tunneling instability [9].

theory	closed strings	D1-brane open strings	D0-brane matrix(-vector) model
bos/Ω_-	T	$\mathrm{Sp}(1)$, $\mathbf{1}$	$\mathrm{Sp}(\frac{n_0}{2})$, $\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2\square$
0B/$\Omega_\pm$	T	–	$\mathrm{SO}(n_0)$, $\begin{smallmatrix}\square\\ \square\end{smallmatrix}_b$ $\mathrm{Sp}(\frac{n_0}{2})$, $\square\!\square_b$
0B/$\widehat{\Omega}_-$	T, C_0	$\mathrm{U}(2)$, $\mathbf{1}_b^0$ $\mathrm{U}(1)^2$, $2(\pm,\mp)_f$	$\mathrm{Sp}(\frac{n_0}{2})$, $\begin{smallmatrix}\square\\ \square\end{smallmatrix}_b + \begin{cases} 4\square_{b\,\mathrm{or}\,f} \\ 2\square_b + 2\square_f \end{cases}$
0A/Ω_-	T, C_1^+	$\mathrm{SO}(2)$, $\mathbf{1}_b$ $-$, $\mathbf{1}_f$	$\mathrm{U}(n_0)$, $\begin{smallmatrix}\square\\ \square\end{smallmatrix}_b + \begin{cases} 2[\square + \bar{\square}]_{b\,\mathrm{or}\,f} \\ [\square + \bar{\square}]_b + [\square + \bar{\square}]_f \end{cases}$
0A/$\widehat{\Omega}_-$	T, C_1^-	$\mathrm{Sp}(1)$, $\mathbf{1}_b$	$\mathrm{Sp}(\frac{n_0}{2}) \times \mathrm{Sp}(\frac{\bar{n}_0}{2})$, $(\square,\square)_b + 2[(\square,\mathbf{1}) + (\mathbf{1},\square)]_{b\,\mathrm{or}\,f}$

Table 1: Tadpole-free unoriented open and closed string theories in two dimensions and their dual matrix(-vector) models.

A breakthrough was made by Zamolodchikov and Zamolodchikov, who recently constructed the D0-brane state in Liouville theory [10]. In light of this, McGreevy and Verlinde proposed to identify the $c = 1$ matrix model with the quantum mechanics of the open string tachyon on N unstable D0-branes of the corresponding two-dimensional string theory [11]. This conjecture was made precise by the authors of [12]. The equivalence of the $c = 1$ matrix model and two-dimensional string theory is now understood as a holographic open/closed string duality, similar to AdS/CFT. It is also related to another recent development in string theory, namely the condensation of open string tachyons (for a review see [13]), and the rolling tachyon background [14]. It is remarkable to see two recent eminent ideas blend into a paradigm, albeit in the realm of a simplified model of string theory.

In a further development, a new interpretation of the $c = 1$ matrix model was proposed in [15, 16]. When the potential of the matrix model is filled symmetrically, it is conjectured to be dual to the two-dimensional type 0B string theory. This avoids the tunneling instability of [9], leading to an unambiguously defined non-perturbative string theory. Many other recent developments have appeared in [17]–[30].

In this paper, we will extend these conjectures to unoriented open and closed string theory. We will classify the possible unoriented bosonic and type 0 string theories in two dimensions (the former was already studied in [31]), solve the tadpole cancellation condition for each one, and construct the corresponding matrix(-vector) models using D0-branes. Our results for the tadpole-free theories are summarized in table 1. More general theories can be found in table 2, and the corresponding matrix models in table 3.

The paper is organized as follows. In section 2 we review the unoriented bosonic string. In section 3 we classify all possible two-dimensional unoriented open and closed type 0 strings. In section 4 we derive the corresponding matrix models by analyzing the D0-branes in each theory, and section 5 contains our conclusions.

As this paper was being written, the paper [32] appeared, with which there is some overlap.

2. Unoriented bosonic string

The two-dimensional unoriented bosonic string was analyzed recently in [31]. Let us review this model and clarify some points. As is well known, gauging the world-sheet parity symmetry of the 26 dimensional critical bosonic string theory gives an unoriented string theory, which has a massless tadpole (for the dilaton and graviton) on RP^2. The easiest way to compute this tadpole is from the one loop Klein bottle amplitude. Channel duality relates this to a tree amplitude involving the square of the RP^2 tadpole. This tadpole can be cancelled against a disk tadpole by introducing open strings. By comparing the Klein bottle, Möbius strip and cylinder amplitudes one finds that cancellation of the massless tadpole requires an open string Chan-Paton gauge group $\mathrm{SO}(2^{13})$. However one is still faced with the problem posed by the closed string tachyon.

In two dimensions the situation is somewhat better, in that the ground state is massless (though we continue to call it a tachyon), and in fact is the only propagating state. It is the tadpole of this state which will concern us. In non-critical string theory one of the dimensions is a Liouville field, which complicates the computation of the one loop amplitudes due to the presence of the bulk and boundary Liouville interactions (cosmological constants). However, the massless tadpoles can be obtained from the one-loop amplitudes in the *free* field theory, which are easily computable. This is because the massless tadpole in Liouville theory corresponds to the IR pole of the corresponding one-point function (on the disk or RP^2), and the residue is given by the one-point function in the free theory [33, 34]. A heuristic way to illustrate this is to look at just the Liouville zero mode, and treat the bulk interaction perturbatively. In this approximation the one-point function on a Riemann surface with Euler number χ is given by

$$\begin{aligned} \langle e^{\alpha\phi}\rangle_\chi &= \int_{-\infty}^{\infty} d\phi_0\, e^{\alpha\phi_0} e^{-\chi Q\phi_0 - \mu e^{2b\phi_0} - \mu_B e^{b\phi_0}} \\ &= \frac{1}{b}\mu^{-\frac{1}{2b}(\alpha-\chi Q)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{\mu}{\mu_B^2}\right)^{\frac{\alpha-\chi Q}{2b}+k} \Gamma\left(\frac{\alpha-\chi Q}{b}+2k\right), \end{aligned} \tag{2.1}$$

where $\alpha = Q+2iP$. The disk and crosscap (RP^2) both have $\chi = 1$, so the massless tadpole corresponds to the $P \to 0$ pole of the Gamma function at the $k = 0$ order. The residue of this pole is independent of μ and μ_B, and corresponds to the free field result.

The Klein bottle, cylinder and Möbius amplitudes (per unit volume) found in [31] are (in the tree channel)

$$\mathcal{A}_K = \int_0^\infty \frac{ds}{8\pi^3} \tag{2.2}$$

$$\mathcal{A}_C = n^2 \int_0^\infty \frac{ds}{32\pi^3} \tag{2.3}$$

$$\mathcal{A}_M = \pm n \int_0^\infty \frac{ds}{8\pi^3}\,, \tag{2.4}$$

where the sign in the Möbius amplitude refers to the choice we have on how Ω acts on the CP factor. We will denote this choice by $\Omega_\pm$, in correspondence with the sign of the RP^2

tachyon tadpole. The corresponding CP groups are $\mathrm{SO}(n_1)$ in the Ω_+ case, and $\mathrm{Sp}(n_1/2)$ in the Ω_- case.[1] Since the open string ground state is even under world-sheet parity Ω, the massless open string tachyon is a symmetric matrix in the orthogonal case and a skew-symmetric matrix in the symplectic case. The latter corresponds to the antisymmetric representation of the symplectic group. In the tadpole-free case we therefore get Sp(1) with a tachyon in the **1**.

This is also consistent with what is obtained in the exact Liouville calculation [31], using the explicit form of the Liouville crosscap state derived in [35].

3. Unoriented type O strings

3.1 Review of critical type 0 strings

Let us first review the critical type 0 string theories and their orientifolds. Many of the properties we will review are shared by the two-dimensional type 0 theories. The critical type 0 string theories are ten-dimensional modular invariant closed fermionic string theories, which consist of an NS-NS and R-R sector only, with a diagonal GSO projection:

$$\begin{aligned} \text{type 0A:} \quad & (\mathrm{NS}+,\mathrm{NS}+) + (\mathrm{NS}-,\mathrm{NS}-) + (\mathrm{R}+,\mathrm{R}-) + (\mathrm{R}-,\mathrm{R}+) \\ \text{type 0B:} \quad & (\mathrm{NS}+,\mathrm{NS}+) + (\mathrm{NS}-,\mathrm{NS}-) + (\mathrm{R}+,\mathrm{R}+) + (\mathrm{R}-,\mathrm{R}-)\,. \end{aligned}$$

As such, they are not spacetime supersymmetric, and in fact possess a tachyon in the $(\mathrm{NS}-,\mathrm{NS}-)$ sector. The type 0 theories also contain two sets of massless R-R gauge fields, C_p^+ and C_p^- (these are even and odd linear combinations of the fields in the above sectors), and correspondingly two sets of R-R charged D-branes, $D(p+1)_{\eta=\pm}$, with p even in type 0B and odd in type 0A [36, 37, 38]. As in the type-II theories, the type 0 theories also possess uncharged and unstable D-branes of the "wrong" dimensions. They also come in two varieties, and are denoted $\widetilde{Dp}_\eta$. The corresponding D-brane boundary states are given by

$$\begin{aligned} |D(p+1)_\eta\rangle &= \frac{1}{\sqrt{2}}\left(|B(p+1),\eta\rangle_{\mathrm{NSNS}} + |B(p+1),\eta\rangle_{\mathrm{RR}}\right) \\ |\widetilde{Dp}_\eta\rangle &= |Bp,\eta\rangle_{\mathrm{NSNS}}\,. \end{aligned} \tag{3.1}$$

It follows immediately from channel duality (appendix A) that open strings between D-branes of the same sign η are spacetime bosons, and those between D-branes of opposite sign are spacetime fermions. Furthermore, the open string spectrum of the charged D-branes contains only GSO-even states, whereas that of the neutral D-branes contains both GSO-even and GSO-odd states (and therefore a tachyon in the same sign case).

A number of different unoriented string theories can be obtained from the type 0 theories [39, 38]. These correspond to gauging the discrete symmetries Ω, $\widehat{\Omega} = \Omega\cdot(-1)^{F_L}$, where F_L is the left-moving part of the spacetime fermion number (or NSR parity), and

[1] Note that this is the opposite convention to the one usually used in critical string theory. There the sign of $\Omega_\pm$ refers to the sign of the RP^2 dilaton tadpole (and also the RR tadpole for the superstring), which is the opposite of the RP^2 tachyon tadpole.

$\Omega' = \Omega \cdot (-1)^f$, where f is the left-moving world-sheet fermion number. In type 0B all three symmetries are involutions, and in type 0A the first two are involutions, whereas Ω' generates a Z_4 symmetry. This gives five distinct models: $0\mathrm{B}/\Omega$, $0\mathrm{B}/\widehat{\Omega}$, $0\mathrm{B}/\Omega'$, $0\mathrm{A}/\Omega$ and $0\mathrm{A}/\widehat{\Omega}$. The sixth possibility, $0\mathrm{A}/\Omega'$, is actually equivalent to one of the type 0B models.

The models differ in their perturbative and D-brane spectra, as well as in the RP^2 tadpoles they possess. By channel duality for the Klein bottle (table 6) we can immediately see that because of the diagonal GSO projection $(1 + (-1)^{f+\tilde{f}})$ in the loop channel, the Ω models have an NS-NS tadpole, but no R-R tadpole. The same is true for the $\widehat{\Omega}$ models, since the effect of $(-1)^{F_L}$ is just to change the sign of the R-R contribution in the trace. On the other hand, the effect of $(-1)^f$ is to change the diagonal GSO projection to $((-1)^f + (-1)^{\tilde{f}})$, therefore the Ω' model possesses an R-R tadpole, but no NS-NS tadpole.[2] Only this model *requires* the addition of open strings for consistency, resulting in the Chan-Paton gauge group U(32). However this introduces NS-NS disk tadpoles, including a tachyon tadpole, as well as a massless (dilaton) tadpole.

We can determine the states of the RP^2 NS-NS tadpoles in the Ω and $\widehat{\Omega}$ models using channel duality. From the action of Ω on the NS-NS and R-R ground states of type 0B [40],

$$\begin{aligned} \Omega|0\rangle_{\mathrm{NS}} \otimes |0\rangle_{\mathrm{NS}} &= |0\rangle_{\mathrm{NS}} \otimes |0\rangle_{\mathrm{NS}} \\ \Omega|S^\alpha\rangle_R \otimes |\widetilde{S}^\beta\rangle_R &= -|S^\beta\rangle_R \otimes |\widetilde{S}^\alpha\rangle_R\,, \end{aligned} \tag{3.2}$$

we see that the Klein bottle amplitude has the form

$$\langle C,+|\Delta|C,-\rangle_{\mathrm{NSNS}} \mp \langle C,+|\Delta|C,+\rangle_{\mathrm{NSNS}} \tag{3.3}$$

in the $0\mathrm{B}/\Omega$ and $0\mathrm{B}/\widehat{\Omega}$ theories, respectively. The tachyon contribution cancels in the first case and adds in the second case. Therefore the $0\mathrm{B}/\Omega$ model has a dilaton tadpole but no tachyon tadpole, whereas the $0\mathrm{B}/\widehat{\Omega}$ model has a tachyon tadpole but no dilaton tadpole. In type 0A the action of Ω on the R-R ground states is

$$\Omega|S^\alpha\rangle_R \otimes |\widetilde{S}^{\dot\beta}\rangle_R = -|S^{\dot\beta}\rangle_R \otimes |\widetilde{S}^\alpha\rangle_R\,, \tag{3.4}$$

or in other words

$$\Omega : C_p^\pm \to \pm C_p^\pm\,. \tag{3.5}$$

The contributions of C_p^+ and C_p^- therefore cancel in the R-R trace, and the Klein bottle in type 0A is just

$$\langle C,+|\Delta|C,-\rangle_{\mathrm{NSNS}}\,. \tag{3.6}$$

The 0A Ω and $\widehat{\Omega}$ models therefore have both a tachyon and a dilaton tadpole.

The main problem with these models, as with the oriented type 0 strings, is the presence of the tachyon (except in the Ω' model). Ignoring this for the moment (or assuming the tachyon has rolled down to its true vacuum), we are faced with massless NS-NS tadpoles in $0\mathrm{B}/\Omega$, $0\mathrm{A}/\Omega$ and $0\mathrm{A}/\widehat{\Omega}$. One has two options here: cancel the NS-NS tadpole by adding open strings (in such a way as not to introduce a R-R tadpole), or employ the Fischler-Susskind mechanism. The first option yields the CP gauge groups $(\mathrm{SO}(n) \times \mathrm{SO}(32-n))^2$ and $\mathrm{SO}(n) \times \mathrm{SO}(32-n)$ for the 0B and (both) 0A models, respectively.

[2]Interestingly, this theory is tachyon-free, since the tachyon is odd under $(-1)^f$, and therefore under Ω'.

3.2 Two-dimensional type 0 strings

Two-dimensional type 0 strings were described in [15, 16]. The relevant two-dimensional CFT combines a free $N = 1$ SCFT corresponding to time, and an $N = 1$ super-Liouville theory corresponding to space. The latter is given by

$$S_L = \frac{1}{4\pi} \int d^2z \left(\partial\phi\bar{\partial}\phi + \psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi} + 2i\mu_0 b^2 \psi\bar{\psi} e^{b\phi} + \mu_0^2 b^2 e^{2b\phi} \right), \tag{3.7}$$

and there is an implicit background charge given by $Q\chi$, where χ is the Euler number of the world-sheet, and Q is given by

$$Q = b + \frac{1}{b}. \tag{3.8}$$

To get two dimensions we take the limit $b \to 1$, keeping $\mu = \mu_0\gamma((1+b^2)/2)/4$ fixed, where $\gamma(x) = \Gamma(x)/\Gamma(1-x)$. The resulting two-dimensional string theories have no propagating physical excitations,[3] and the only propagating fields are the ground states. In the NS-NS sector this is the tachyon, which is actually massless due to the background charge. The R-R sector contains a single massless scalar C_0 (combining the self-dual and anti-self-dual parts) in the type 0B case, and a pair of massless vectors $C_1^{\pm}$ in the type 0A case.[4] The considerations of world-sheet fermion spin structures are basically the same as in the ten-dimensional case, so many of the results we described above hold also in the two-dimensional case. The main difference (in the closed string case) is that the Liouville interaction breaks the global $(-1)^f$ symmetry.[5] Consequently the $\Omega' = \Omega \cdot (-1)^f$ model does not exist in two-dimensions.[6] We are left with just the two Ω models, and the two $\widehat{\Omega}$ models.

Open strings can be incorporated by including boundaries with either Neumann or Dirichlet boundary conditions for the matter and super-Liouville fields. The problem of a Neumann boundary condition for the Liouville field was solved in [33, 43], and the Dirichlet case was solved in [10]. These were later extended to the super-Liouville theory in [44, 45]. This requires the addition of a boundary Liouville action [16]

$$S_{\partial L} = \frac{1}{2\pi} \oint dx \left(\gamma\partial_x\gamma - \mu_B b\gamma(\psi + \eta\bar{\psi}) e^{b\phi/2} + \mu_B^2 e^{b\phi} \right), \tag{3.9}$$

where γ is a boundary fermionic field.

The Liouville Neumann boundary state is labeled by a parameter ν, which is defined by

$$\cosh^2(\pi b\nu) = \frac{\mu_B^2}{2\mu_0} \cos\left(\frac{\pi b^2}{2}\right), \tag{3.10}$$

and can have four possible spin structures labeled by η and η', where $\eta' = \pm 1$ corresponds to the R-R and NS-NS sector, respectively. In terms of the Ishibashi states $|B, P, \eta\rangle_{\eta'}$, the

[3]There are however discrete physical states [41] (see also [42]).

[4]There is an imaginary shift ib of the background charge in the R-R sector due to the presence of the fermionic zero modes.

[5]This is why it cannot be gauged, for $\mu_0 \neq 0$, to yield two-dimensional type-II strings.

[6]We thank Jaume Gomis for pointing this out to us.

JHEP01(2004)043

theory	closed strings	open string CP group	open string matter	T tadpole
bos/$\Omega_\pm$	T	$\mathrm{SO}(n_1)$	$\Box\!\Box$	n_1+2
		$\mathrm{Sp}(\frac{n_1}{2})$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	n_1-2
0B/$\Omega_\pm$	T	$\mathrm{SO}(n_1^+)^2\times\mathrm{SO}(n_1^-)^2$	$(n_1^\pm,n_1^\pm)_b+(n_1^\pm,n_1^\mp)_f$	$2n_1$
		$\mathrm{Sp}(\frac{n_1^+}{2})^2\times\mathrm{Sp}(\frac{n_1^-}{2})^2$	$(n_1^\pm,n_1^\pm)_b+(n_1^\pm,n_1^\mp)_f$	$2n_1$
0B/$\widehat{\Omega}_\pm$	T,C_0	$\mathrm{U}(n_1^+)\times\mathrm{U}(n_1^-)$	$\Box\!\Box_b^\pm+2(n_1^\pm,\bar{n}_1^\mp)_f$	n_1+2
		$\mathrm{U}(n_1^+)\times\mathrm{U}(n_1^-)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}{}_b^\pm+2(n_1^\pm,\bar{n}_1^\mp)_f$	n_1-2
0A/$\Omega_\pm$	T,C_1^+	$\mathrm{Sp}(\frac{n_1^+}{2})\times\mathrm{Sp}(\frac{n_1^-}{2})$	$\Box\!\Box_b^\pm+2(n_1^+,n_1^-)_f$	n_1+2
		$\mathrm{SO}(n_1^+)\times\mathrm{SO}(n_1^-)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}{}_b^\pm+2(n_1^+,n_1^-)_f$	n_1-2
0A/$\widehat{\Omega}_\pm$	T,C_1^-	$\mathrm{SO}(n_1^+)\times\mathrm{SO}(n_1^-)$	$\Box\!\Box_b^\pm+2(n_1^+,n_1^-)_f$	n_1+2
		$\mathrm{Sp}(\frac{n_1^+}{2})\times\mathrm{Sp}(\frac{n_1^-}{2})$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}{}_b^\pm+2(n_1^+,n_1^-)_f$	n_1-2

Table 2: Unoriented open and closed string theories in two dimensions.

Cardy boundary states are given by

$$|B;\nu,\eta\rangle_{\eta'}=\int_0^\infty dP\,\Psi_\eta^{\eta'}(\nu,P)|B;P,\eta\rangle_{\eta'}\,, \tag{3.11}$$

where the different boundary state wave-functions (normalized disk one-point functions) are given by

$$\Psi_\eta^{\mathrm{NS}}(\nu,P)\;=\;(2\mu)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-\pi iP}\cos(2\pi P\nu)\,, \tag{3.12}$$

$$\Psi_{\eta=+\,\mathrm{sgn}(\mu)}^{\mathrm{RR}}(\nu,P)\;=\;(2\mu)^{-iP/b}\frac{\Gamma(\frac{1}{2}+iPb)\Gamma(\frac{1}{2}+iP/b)}{\sqrt{2}\pi}\cos(2\pi P\nu)\,, \tag{3.13}$$

$$\Psi_{\eta=-\,\mathrm{sgn}(\mu)}^{\mathrm{RR}}(\nu,P)\;=\;(2\mu)^{-iP/b}\frac{\Gamma(\frac{1}{2}+iPb)\Gamma(\frac{1}{2}+iP/b)}{\sqrt{2}\pi}\sin(2\pi P\nu)\,. \tag{3.14}$$

Therefore type 0B contains two kinds of R-R charged D1-brane $D1_\pm$, and type 0A contains two kinds of neutral D1-brane $\widetilde{D1}_\pm$, as in (3.1). The open string spectrum has no physical states for two type 0B D1-branes of the same sign (since the tachyon is GSO-projected out), and a single massless fermion for two D1-branes of the opposite sign. For the type 0A D1-branes, the physical open string spectrum contains a massless tachyon in the same sign case, and a pair of massless fermions in the opposite sign case (no GSO-projection).

We defer the discussion of the Dirichlet boundary states, i.e. the D0-branes, to the next section, where we will construct the matrix models. Let us next analyze the four possible unoriented models in turn. We will apply what we have learned in the ten-dimensional models using channel duality of the world-sheet fermionic spin structures, and also verify the results by explicit computation of the relevant amplitudes in the two-dimensional models. The results are summarized in table 2.

JHEP01(2004)043

3.3 $0B/\Omega$

The R-R scalar C_0 is odd under Ω, and the tachyon T is even, so the tachyon is the only propagating degree of freedom in this theory. The non-dynamical R-R two-forms $C_2^{\pm}$ are even, so both $D1_\eta$-branes and $\overline{D1}_\eta$-branes are invariant. Exactly the same world-sheet spin structure considerations as in the ten-dimensional case show that there is no RP^2 R-R tadpole, and that there is no tachyon tadpole.[7] We can also demonstrate this by an explicit computation of the Klein bottle amplitude in the *free* field theory:[8]

$$\begin{aligned}\mathcal{A}_K &= \frac{1}{2}\int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}_{\mathrm{NSNS+RR}}\left(\frac{1+(-1)^{f+\tilde{f}}}{2}\Omega\, e^{-2\pi t(L_0+\widetilde{L}_0)}\right)\\ &= \frac{1}{2}\int_0^\infty \frac{dt}{2t}\int\frac{d^2p}{(2\pi)^2}e^{-\frac{\pi t}{2}(p_0^2+p_1^2)}\,\mathrm{Tr}_{\mathrm{NSNS+RR}}\left(\frac{1+(-1)^{f+\tilde{f}}}{2}\Omega\right).\end{aligned} \tag{3.15}$$

Once we ignore the Liouville interaction the actual calculation becomes trivial, since all the excited states in two-dimensional string theory are longitudinal (except for non-propagating discrete states) and thus cancelled out by the ghost contributions. After the usual modular transformation to the tree channel $s = \pi/2t$ we find

$$\mathcal{A}_K = \frac{2}{(2\pi)^3}\int_0^\infty ds\,(1-1) = \int_0^\infty ds\,\langle T\rangle_{O1}^2\,, \tag{3.16}$$

and therefore $\langle T\rangle_{O1} = 0$. This model is therefore tadpole-free without open strings.

Nevertheless, one can consider adding open strings by including $D1_\eta\overline{D1}_\eta$ pairs. This does not introduce a net R-R charge, so the theory is consistent, but it will have a disk tachyon tadpole. We can compute this tadpole from the NS-NS exchange part of the cylinder amplitude. Channel duality tells us that this is

$$\mathcal{A}_C^{\mathrm{NSNS}} = \frac{n_1^2}{2}\int_0^\infty\frac{dt}{2t}\,\mathrm{Tr}_{\mathrm{NS}}\,\frac{1}{2}e^{-2\pi t L_0} = \frac{n_1^2}{4}\int_0^\infty\frac{dt}{2t}\int\frac{d^2p}{(2\pi)^2}e^{-\pi t(p_0^2+p_1^2)}\,, \tag{3.17}$$

where again the free field computation reduces to just the massless tachyon contribution. In the tree channel this becomes (with $s = \pi/t$)

$$\mathcal{A}_C^{\mathrm{NSNS}} = \frac{n_1^2}{4(2\pi)^3}\int_0^\infty ds = \int_0^\infty ds\,\langle T\rangle_{D1_\eta}^2\,, \tag{3.18}$$

and so $\langle T\rangle_{D1_\eta} = n_1/(2(2\pi)^{3/2})$. For completeness we also present the Möbius strip amplitude:

$$\begin{aligned}\mathcal{A}_M &= \frac{1}{2}\int_0^\infty\frac{dt}{2t}\,\mathrm{Tr}_{\mathrm{NS}}\left(\frac{1}{2}\left(1+(-1)^f\right)\Omega\, e^{-2\pi t L_0}\right)\\ &= \pm n_1\,\frac{1}{2}\int_0^\infty\frac{dt}{2t}\int\frac{d^2p}{(2\pi)^2}e^{-\pi t(p_0^2+p_1^2)}(-i)\frac{1}{2}(1-1)\,.\end{aligned} \tag{3.19}$$

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[7] This was originally pointed out to us independently by Jaume Gomis.

[8] Our convention for the open string momentum is fixed by $L_0 = \frac{1}{2}\left(p_0^2+p_1^2\right)+\cdots$, while the closed string momentum (squared) differs by a factor of $\frac{1}{4}$, thus $L_0 = \frac{1}{8}(p_0^2+p_1^2)+\cdots$ and $\widetilde{L}_0 = \frac{1}{8}(p_0^2+p_1^2)+\cdots$.

JHEP01(2004)043

The $-i$ comes from the action of Ω on the NS ground state. In the tree channel with $s = \pi/4t$ this becomes

$$\mathcal{A}_M = \pm \frac{2n_1}{(2\pi)^3} \int_0^\infty ds\, (-i) \frac{1}{2}(1-1) = 2 \int_0^\infty ds\, \langle T \rangle_{D1_\eta} \langle T \rangle_{O1} \,. \tag{3.20}$$

The open string spectrum for n_1^+ $D1_+\overline{D1}_+$ pairs and n_1^- $D1_-\overline{D1}_-$ pairs contains massless tachyons in the bi-fundamentals $(n_1^+, n_1^+) + (n_1^-, n_1^-)$ of either $\mathrm{SO}(n_1^+)^2 \times \mathrm{SO}(n_1^-)^2$ or $\mathrm{Sp}(n_1^+/2)^2 \times \mathrm{Sp}(n_1^-/2)^2$, and massless fermions in the bi-fundamentals $2(n_1^+, n_1^-)$.

3.4 0B/$\widehat{\Omega}$

As in ten-dimensions, this model does not have a R-R tadpole either. It does however have a massless tachyon tadpole, which we will compute below. In this case C_0 is even and $C_2^\pm$ is odd, since $(-1)^{F_L}$ gives an additional minus sign for all R-R fields. Therefore both T and C_0 survive in the closed string spectrum. On the other hand, since $C_2^\pm$ is odd, $\widehat{\Omega}$ interchanges the $D1_\eta$-brane and the $\overline{D1}_\eta$-brane. The invariant $D1_\eta\overline{D1}_\eta$ combination is neutral, so we denote it $\widetilde{D1}_\eta$, by analogy with the "wrong" dimension neutral D-branes. The open string states on a single $\widetilde{D1}_\eta$ can be represented in terms of U(2) Chan-Paton factors as

$$\begin{aligned}
11 + \bar{1}\bar{1} \text{ strings}: \quad |p; N; \mathbb{I}\rangle &= \frac{1}{\sqrt{2}} \mathbb{I}_{ij}\, |p; N; ij\rangle \\
1\bar{1} + \bar{1}1 \text{ strings}: \quad |p; N; \sigma_1\rangle &= \frac{1}{\sqrt{2}} (\sigma_1)_{ij}\, |p; N; ij\rangle \\
1\bar{1} - \bar{1}1 \text{ strings}: \quad |p; N; \sigma_2\rangle &= \frac{1}{\sqrt{2}} (\sigma_2)_{ij}\, |p; N; ij\rangle \\
11 - \bar{1}\bar{1} \text{ strings}: \quad |p; N; \sigma_3\rangle &= \frac{1}{\sqrt{2}} (\sigma_3)_{ij}\, |p; N; ij\rangle \,.
\end{aligned} \tag{3.21}$$

For n_1 $\widetilde{D1}_\eta$-branes we then have four sectors, labeled by the $2n_1 \times 2n_1$ matrices $\Lambda = \{\mathbb{I}, \Sigma_1, \Sigma_2, \Sigma_3\}$, where $\Sigma_i = \mathbb{I} \otimes \sigma_i$.

Let us now compute the relevant one-loop amplitudes. The cylinder amplitude for n_1 $\widetilde{D1}_\eta$-branes is given by

$$\mathcal{A}_C = \frac{n_1^2}{2} \int_0^\infty \frac{dt}{2t} \int \frac{d^2p}{(2\pi)^2} \mathrm{Tr}_{\mathrm{NS}} \left(P_{\mathrm{GSO}}^+ + P_{\mathrm{GSO}}^- + P_{\mathrm{GSO}}^- + P_{\mathrm{GSO}}^+\right) e^{-2\pi t L_0} \,, \tag{3.22}$$

where we have explicitly included the sum over the four sectors with the appropriate GSO-projections; the $\mathbb{I}$ and Σ_3 sectors contain only GSO-even states, and the Σ_1 and Σ_2 sectors contain only GSO-odd states. This reduces to twice the amplitude for unprojected open NS strings

$$\mathcal{A}_C = 2 \cdot \frac{n_1^2}{2} \int_0^\infty \frac{dt}{2t} \int \frac{d^2p}{(2\pi)^2} e^{-\pi t (p_0^2 + p_1^2)} = \frac{n_1^2}{(2\pi)^3} \int_0^\infty ds = n_1^2 \int_0^\infty ds\, \langle T \rangle^2_{\widetilde{D1}_\eta} \,, \tag{3.23}$$

which reflects the fact that there is no R-R exchange in the closed string channel, and that the "tension" of the $\widetilde{D1}_\eta$ is twice that of the $D1_\eta$.[9] The Möbius amplitude is

$$\mathcal{A}_M = \frac{n_1}{2}\int_0^\infty \frac{dt}{2t}\int \frac{d^2p}{(2\pi)^2}\,\mathrm{Tr}_{\mathrm{NS}}\left(P^+_{\mathrm{GSO}} \pm P^-_{\mathrm{GSO}} \pm P^-_{\mathrm{GSO}} - P^+_{\mathrm{GSO}}\right)\Omega e^{-2\pi t L_0}\,, \tag{3.24}$$

where we have included the two possible actions of $\widehat{\Omega} = \Omega\cdot(-1)^{F_L}$ on the different CP factors (so Ω here acts only on the Virasoro states). The signs are explained as follows. Recall that the operator $(-1)^{F_L}$ exchanges the D1-brane and anti-D1-brane, so its action is equivalent to conjugation by Σ_1. The action of Ω on the CP factors has two possibilities:

$$\Omega : \Lambda \mapsto \begin{cases} \Lambda^T \\ \Sigma_3\Lambda^T\Sigma_3\,. \end{cases} \tag{3.25}$$

This follows from the requirement that $\widehat{\Omega}^2 = 1$, and the fact that the D1-brane and anti-D1-brane are each invariant under Ω. Combining this with the action of $(-1)^{F_L}$ we get

$$\widehat{\Omega} : \Lambda \mapsto \begin{cases} \Sigma_1\Lambda^T\Sigma_1 \\ \Sigma_2\Lambda^T\Sigma_2\,, \end{cases} \tag{3.26}$$

and hence the corresponding signs in (3.24). The amplitude then reduces to

$$\begin{aligned}
\mathcal{A}_M &= \pm 2\cdot\frac{n_1}{2}\int_0^\infty \frac{dt}{2t}\int\frac{d^2p}{(2\pi)^2}\,\mathrm{Tr}_{\mathrm{NS}}\,P^-_{\mathrm{GSO}}\Omega e^{-2\pi t L_0} \\
&= \pm n_1 \int_0^\infty \frac{dt}{2t}\int\frac{d^2p}{(2\pi)^2}\,e^{-\pi t(p_0^2+p_1^2)} \\
&= \pm\frac{4n_1}{(2\pi)^3}\int_0^\infty ds = 2n_1\int_0^\infty ds\,\langle T\rangle_{\widetilde{D1}_\eta}\langle T\rangle_{\widehat{O1}}\,,
\end{aligned} \tag{3.27}$$

where in the third equality we used the modular transformation $s = \pi/4t$. We conclude that

$$\langle T\rangle_{\widehat{O1}} = \pm 2\langle T\rangle_{\widetilde{D1}_\eta}\,, \tag{3.28}$$

and denote the two models by $\widehat{\Omega}_\pm$. The tachyon tadpole can only be cancelled for $\widehat{\Omega}_-$. The Klein bottle amplitude can be computed in a similar way, but we leave it out as it provides no new information.

The open string spectrum can be read off from (3.26), and the known action of Ω on the ground states. Let us first consider only one type of $\widetilde{D1}_\eta$-brane, say $\widetilde{D1}_+$. Of the two GSO-even sectors only one survives, so the symmetry is $\mathrm{U}(n_1)$ for both $\widehat{\Omega}_\pm$. In the tadpole-free case we therefore have U(2). The GSO-odd sectors are both even under $\widehat{\Omega}_+$, and both odd under $\widehat{\Omega}_-$. Since the tachyon wave-function is even the tachyon is in the symmetric representation in the first case, and in the antisymmetric representation in the second case. In the tadpole-free case we therefore have a U(2) singlet tachyon.

[9] This is different than the tension of a "wrong" dimension brane, which is always $\sqrt{2}$ times the tension of the same dimension brane in the other theory.

By including both types of 1-brane we also get massless fermions. For n_1^+ $\widetilde{D1}_+$s and n_1^- $\widetilde{D1}_-$s the "gauge group" is $\mathrm{U}(n_1^+) \times \mathrm{U}(n_1^-)$, the tachyons are in the symmetric and antisymmetric representations for $\widehat{\Omega}_+$ and $\widehat{\Omega}_-$ case, respectively, and the fermions are in (two copies of) the bi-fundamental representation (and its conjugate). In particular, we have another tadpole-free theory with $n_1^+ = n_1^- = 1$, that is with a CP group $\mathrm{U}(1) \times \mathrm{U}(1)$, and two charged massless fermions plus their complex conjugates.

3.5 0A/Ω and 0A/$\widehat{\Omega}$

As in ten dimensions, both these models produce a tachyon tadpole, but no R-R tadpole. The action of Ω interchanges the (R$-$, R$+$) and (R$+$, R$-$) sectors, so that C_1^+ is even and C_1^- is odd. Precisely the opposite holds for the action of $\widehat{\Omega}$, since it includes also $(-1)^{F_L}$, namely C_1^+ is odd and C_1^- is even. The massless NS-NS tachyon is even in both cases.

Let us begin with the Klein bottle amplitude. Note that the R-R sector does not contribute to this amplitude in the loop channel, for either Ω or $\widehat{\Omega}$, in type 0A; the contributions of C^+ and C^- cancel. We therefore find

$$\begin{aligned}\mathcal{A}_K &= \frac{1}{2}\int_0^\infty \frac{dt}{2t}\int \frac{d^2p}{(2\pi)^2} e^{-\frac{\pi t}{2}(p_0^2+p_1^2)}\,\mathrm{Tr}_{\mathrm{NSNS}}\left(\frac{1+(-1)^{f+\tilde{f}}}{2}\Omega\right)\\ &= \frac{2}{(2\pi)^3}\int_0^\infty ds = \int_0^\infty ds \langle T\rangle_{O1}^2\,, \end{aligned} \tag{3.29}$$

and exactly the same for $\widehat{\Omega}$. We see that there is a tachyon tadpole in both models (and no R-R tadpole, due to the diagonal GSO-projection).

Now consider adding $\widetilde{D1}_\eta$-branes. The open strings between like-sign branes are NS strings. There are two CP sectors, corresponding to the $\mathbb{I}$ and σ_1 sectors of the $D1_\eta\overline{D1}_\eta$ pair in type 0B (3.21). This follows from Sen's construction of the $\widetilde{Dp}$-brane in type IIA(B) as a projection of a $Dp\overline{Dp}$ pair in type IIB(A) by $(-1)^{F_L}$ [13]. The projection removes the σ_2 and σ_3 sectors. The cylinder amplitude is therefore given by the amplitude for unprojected NS strings

$$\mathcal{A}_C = \frac{n_1^2}{2}\int_0^\infty \frac{dt}{2t}\int \frac{d^2p}{(2\pi)^2} e^{-\pi t(p_0^2+p_1^2)} = \frac{n_1^2}{2(2\pi)^3}\int_0^\infty ds = n_1^2\int_0^\infty ds\,\langle T\rangle_{\widetilde{D1}_\eta}^2\,. \tag{3.30}$$

To compute the Möbius amplitude we first need to determine how Ω and $\widehat{\Omega}$ act on the open string sectors. The action on the CP factors is standard

$$\Omega\,,\widehat{\Omega} : \Lambda \mapsto \Gamma\Lambda^T\Gamma^{-1}\,, \qquad \text{where } \Gamma = \mathbb{I} \text{ or } \Sigma_2\,. \tag{3.31}$$

The action on the open string states is then determined by the action on the ground states of the different sectors. In the GSO-even $\mathbb{I}$ sector the action is standard

$$\Omega, \widehat{\Omega}|p;0;\mathbb{I}\rangle = -i|p;0;\mathbb{I}\rangle\,, \tag{3.32}$$

so the would-be vector, which is the first excited state (and the lowest to survive the GSO projection) is odd under Ω and $\widehat{\Omega}$. The CP group (which is now not really a gauge group, since there is no gauge field) in both models is therefore $SO(n_1)$ for $\Gamma = \mathbb{I}$ and $Sp(n_1/2)$ for $\Gamma = \Sigma_2$. The action on the GSO-odd sector ground state, i.e. the massless tachyon, can be determined using Sen's argument [13]: since the $\widetilde{D1}_\eta$-brane world-sheet theory contains a term

$$\int C_1 \wedge dt\,, \tag{3.33}$$

the transformation property of the open string tachyon t must be the same as that of C_1. But which C_1 is this? As we will review in the next section, only one type of D0-brane is consistent in two-dimensional type 0A string theory, $D0_-$ in our conventions. Since the above term identifies a tachyonic kink as a source for the R-R field C_1, i.e. a D0-brane, the R-R field must be C_1^-. It follows that the tachyon is odd under Ω and even under $\widehat{\Omega}$,

$$\Omega|p;0;\Sigma_1\rangle \;=\; -|p;0;\Sigma_1\rangle \tag{3.34}$$

$$\widehat{\Omega}|p;0;\Sigma_1\rangle \;=\; |p;0;\Sigma_1\rangle\,. \tag{3.35}$$

Therefore the tachyon belongs to the (anti)symmetric representation of Sp(SO) in the Ω model, and to the (anti)symmetric representation of SO(Sp) in the $\widehat{\Omega}$ model. The Möbius amplitudes are given by

$$\mathcal{A}_M \;=\; \pm\frac{n}{2}\int_0^\infty \frac{dt}{2t}\int \frac{d^2p}{(2\pi)^2} e^{-\pi t(p_0^2+p_1^2)} \;=\; \pm\frac{2n_1}{(2\pi)^3}\int_0^\infty ds \tag{3.36}$$

$$\mathcal{A}_{\widehat{M}} \;=\; -\mathcal{A}_M = \mp\frac{2n_1}{(2\pi)^3}\int_0^\infty ds\,, \tag{3.37}$$

and therefore

$$\langle T\rangle_{O1} \;=\; \pm 2\langle T\rangle_{\widetilde{D1}_\eta} \tag{3.38}$$

$$\langle T\rangle_{\widehat{O1}} \;=\; \mp 2\langle T\rangle_{\widetilde{D1}_\eta}\,, \tag{3.39}$$

where in both models the upper sign corresponds to the CP group $Sp(n_1/2)$, and the lower sign to $SO(n_1)$. The two tadpole free models are therefore $0A/\Omega$ with CP group SO(2) and a single tachyon, and $0A/\widehat{\Omega}$ with CP group $Sp(1) = SU(2)$ and a tachyon in the **1**.

In the more general situation where we add both types of $\widetilde{D1}_\eta$-branes we get a product CP group, e.g. $SO(n_1^+) \times SO(n_1^-)$, and also massless fermions in bi-fundamental representations. In particular, we can get one more tadpole free model with $n_1^+ = n_1^- = 1$, no CP group, no massless tachyons, only a single massless fermion.

4. Dual matrix models

In this section we will propose a dual matrix model for each of the unoriented two-dimensional string theories constructed in the previous sections, in terms of the corresponding unstable D0-brane quantum mechanics. To obtain the matrix models for the unoriented theories we need to determine the D0-brane gauge group and open string matter content in each case. This is done using now standard techniques. For each of the four models we will have two choices for the gauge group and/or matter representation, just as

JHEP01(2004)043

theory	D0-brane gauge group	D0-brane matter
bos/$\Omega_\pm$	$\mathrm{SO}(n_0)$	$\square\!\square + n_1 \square$
	$\mathrm{Sp}(\frac{n_0}{2})$	$\begin{smallmatrix}\square\\\square\end{smallmatrix} + n_1 \square$
0B/$\Omega_\pm$	$\mathrm{SO}(n_0)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_b + 2n_1^- \square_b + 2n_1^+ \square_f$
	$\mathrm{Sp}(\frac{n_0}{2})$	$\square\!\square_b + 2n_1^- \square_b + 2n_1^+ \square_f$
0B/$\widehat{\Omega}_\pm$	$\mathrm{SO}(n_0)$	$\square\!\square_b + 2n_1^- \square_b + 2n_1^+ \square_f$
	$\mathrm{Sp}(\frac{n_0}{2})$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_b + 2n_1^- \square_b + 2n_1^+ \square_f$
0A/$\Omega_\pm$	$\mathrm{U}(n_0)$	$\square\!\square_b + n_1^- [\square + \overline{\square}]_b + n_1^+ [\square + \overline{\square}]_f$
	$\mathrm{U}(n_0)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_b + n_1^- [\square + \overline{\square}]_b + n_1^+ [\square + \overline{\square}]_f$
0A/$\widehat{\Omega}_\pm$	$\mathrm{SO}(n_0) \times \mathrm{SO}(\bar{n}_0)$	$(\square,\square)_b + n_1^- [(\square,\mathbf{1}) + (\mathbf{1},\square)]_b + n_1^+ [(\square,\mathbf{1}) + (\mathbf{1},\square)]_f$
	$\mathrm{Sp}(\frac{n_0}{2}) \times \mathrm{Sp}(\frac{\bar{n}_0}{2})$	$(\square,\square)_b + n_1^- [(\square,\mathbf{1}) + (\mathbf{1},\square)]_b + n_1^+ [(\square,\mathbf{1}) + (\mathbf{1},\square)]_f$

Table 3: D0-brane matrix models for unoriented 2d string theories.

in the D1-brane case. What will be slightly less trivial is determining which choice for the D0-branes corresponds to which choice for the D1-branes. Our approach will be to compare the relation between the D1 disk tadpole and the RP^2 tadpole, which we will obtain by computing the D0-D1 cylinder amplitude and the D0-D0 Möbius amplitude, with the corresponding relation obtained in section 3 from the D1-D1 cylinder and Möbius amplitudes. This will fix the D0-brane open string data relative to the D1-brane open string data. The results are summarized in table 3.

4.1 Bosonic string

We begin again with the bosonic string. The D0-brane corresponds to a Liouville Dirichlet boundary state tensored with a $c = 1$ (and ghost) Neumann boundary state. The former is labeled by a pair of integers (n, m), in one-to-one correspondence with the degenerate conformal families $[V_{n,m}]$. The open strings between a $(1, 1)$ D0-brane and an (n, m) D0-brane belong to this family [10]. In particular, the physical open string spectrum on a $(1, 1)$ D0-brane contains only a tachyon (which is now truly tachyonic, since the mass is not shifted). This is the D0-brane used in constructing the matrix model for the oriented bosonic string [12]. The role of the other (n, m) D0-branes is not yet clear.[10] In what follows we will refer to the $(1, 1)$ D0-brane simply as the D0-brane.

It has also been shown that the open strings between the D0-brane and a D1-brane with (continuous) parameter ν correspond to the conformal family $[V_\alpha]$, with $\alpha = Q/2 + i\nu/2$ [10]. The conformal weights of V_α and $V_{n,m}$ are given by $\Delta_\alpha = Q^2/4 - (Q - 2\alpha)^2/4$ and $\Delta_{n,m} = Q^2/4 - (nb + m/b)^2/4$, respectively. In the (n, m) degenerate module, the null state appears at level nm, so its conformal weight is $\Delta_{null} = Q^2/4 - (nb - m/b)^2/4$.

[10] For a recent proposal however see [28, 29] and [30].

The cylinder amplitude for the D0-D1 strings is therefore given by (including also the $c = 1$ matter and ghost contributions):

$$\mathcal{A}_{C,0-1} = n_0 n_1 \int_0^\infty \frac{dt}{2t} \int \frac{dp_0}{2\pi} e^{-\pi t p_0^2} e^{-\frac{\pi t}{2}\nu^2} = \frac{n_0 n_1}{4\pi^{3/2}} \int_0^\infty \frac{ds}{s^{1/2}} e^{-\frac{\pi^2}{2s}\nu^2} , \tag{4.1}$$

where n_0 and n_1 are the numbers of D0-branes and D1-branes, and we have performed the usual loop to tree channel transformation for the cylinder $s = \pi/t$.

Turning our attention to the D0-D0 strings, let us first consider the more general case of (n, m) D0-branes. To compute the cylinder or Möbius amplitude for (n, m) D0-branes one needs to evaluate the trace in the conformal family $[V_{n,m}]$, and subtract the trace in the null module beginning with the null state. For the cylinder this gives

$$\mathcal{A}_{C,0-0} = \frac{n_0^2}{2} \int_0^\infty \frac{dt}{2t} \int \frac{dp_0}{2\pi} e^{-\pi t p_0^2} \left(e^{\frac{\pi t}{2}(nb+m/b)^2} - e^{\frac{\pi t}{2}(nb-m/b)^2} \right) , \tag{4.2}$$

and for the Möbius strip this gives [35]

$$\mathcal{A}_{M,0-0} = \pm n_0 \frac{1}{2} \int_0^\infty \frac{dt}{2t} \int \frac{dp_0}{2\pi} e^{-\pi t p_0^2} \left(e^{\frac{\pi t}{2}(nb+m/b)^2} - (-1)^{nm} e^{\frac{\pi t}{2}(nb-m/b)^2} \right) . \tag{4.3}$$

The relative factor $(-1)^{nm}$ comes from the fact that the null module begins at level nm, and from the action of Ω on the Virasoro generators $\Omega L_n \Omega^{-1} = (-1)^n L_n$. Specializing to the case $b = 1$ (2d string), and $(n, m) = (1, 1)$, and performing the modular transformation $s = \pi/4t$, the Möbius amplitude becomes

$$\mathcal{A}_{M,0-0} = \pm \frac{2n_0}{4\pi^{3/2}} \int_0^\infty \frac{ds}{s^{1/2}} e^{\frac{\pi^2}{4s}} . \tag{4.4}$$

Comparing the large s behavior of this amplitude with the large s behavior of the D0-D1 cylinder amplitude (4.1) we see that the tadpoles are related as $\langle T \rangle_{O1} = \pm 2 \langle T \rangle_{D1}$, in agreement with what we found in section 2 using the free field computations of the D1-brane annulus and Möbius amplitudes.

The upper sign in the Möbius amplitude (or the RP^2 tadpole) corresponds to the D0-brane gauge group SO(n_0), and the lower sign to Sp($n_0/2$), paralleling the choice of the D1-brane CP group. The open string tachyon on the D0-brane is even under Ω, and therefore transforms in the (anti)symmetric representation in the SO(Sp) case. In addition, there are n_1 scalars in the vector representation of the D0-brane gauge group from the D0-D1 strings. Their mass is given by

$$m_{01}^2 = \frac{\nu^2}{4\alpha'} . \tag{4.5}$$

These are tachyonic as well if ν is imaginary. In the tadpole-free case we therefore get an Sp($n_0/2$) matrix model with a symmetric matrix and two vectors.

4.2 Type 0

The Dirichlet boundary state in super-Liouville theory is again labeled by a pair of integers (n, m). In this case the open strings between a $(1, 1)$ state and an (n, m) state can only be

NS if $n-m$ is even, and R if $n-m$ is odd. The open strings between a $(1,1)$ and an (n,m) Dirichlet state, and between a $(1,1)$ Dirichlet state and a ν Neumann state again correspond to the conformal families of $V_{n,m}$ and V_α, respectively, where $\alpha = Q/2 + i\nu/2$. However the conformal weights of $V_{n,m}$ and V_α are now given by $\Delta_{n,m} = Q^2/8 - (nb+m/b)^2/8$ and $\Delta_\alpha = Q^2/8 - (Q-2\alpha)^2/8$ in the NS sector, and by $\Delta_{n,m} = Q^2/8 + 1/16 - (nb+m/b)^2/8$ and $\Delta_\alpha = Q^2/8+1/16-(Q-2\alpha)^2/8$ in the R sector. The null state appears at level $nm/2$ in the (n,m) degenerate module in both the NS and R sectors. As in the bosonic case, we will only be interested in the $(1,1)$ Dirichlet state.

In super-Liouville theory it turns out that the Dirichlet boundary state is consistent only for one value of the spin structure η, namely for $\eta = -\,\mathrm{sgn}(\mu_0)$.[11] This implies that there is only one type of D0-brane in two-dimensional type 0A or type 0B string theory. For definiteness let us fix $\mu_0 > 0$. Then in type 0A there is only a $D0_-$, and in type 0B there is only a $\widetilde{D0}_-$. This is also consistent with the fact that in type 0A the exponential tachyon background given by the Liouville interaction allows only the R-R field strength $F^- = dC_1^-$ to have a non-vanishing time-independent value [16]. It is also consistent with the fact that $(-1)^f$ is not a symmetry. This operator changes the sign of η, and therefore interchanges the two types of D-brane. Had it been a symmetry, both types would have to exist (as in the ten-dimensional theories). Of course the absence of this symmetry does not forbid the presence of the other type of D-brane, as we see in the D1-brane case.

The matrix models for the oriented theories correspond to the quantum mechanics of n_0 $\widetilde{D0}_-$-branes in type 0B, and n_0 $D0_-$-branes plus $\bar{n}_0$ $\overline{D0}_-$-branes in type 0A [16]. The former is therefore the hermitean $\mathrm{U}(n_0)$ matrix model, and the latter is the quiver $\mathrm{U}(n_0)\times\mathrm{U}(\bar{n}_0)$ matrix model. Let us now determine the corresponding matrix models for the unoriented theories.

4.2.1 The 0B models

The $\widetilde{D0}_-$-$\widetilde{D0}_-$ open strings are NS strings, so $n-m$ is restricted to be even. Like the $\widetilde{D1}_\eta$-$\widetilde{D1}_\eta$ strings in type 0A, there are two CP sectors: the GSO-even $\mathbb{I}$ sector, and the GSO-odd Σ_1 sector. The action of Ω and $\widehat{\Omega}$ on the GSO-even sector is the same as in the case of the type 0A $\widetilde{D1}_\eta$-brane, so the gauge group is $\mathrm{SO}(n_0)$ for $\Gamma = \mathbb{I}$, and $\mathrm{Sp}(n_0/2)$ for $\Gamma = \Sigma_2$. To determine the action on the GSO-odd sector we will use Sen's argument again. The $\widetilde{D0}_-$-brane world-line theory contains the term

$$\int C_0 \wedge dt\,, \tag{4.6}$$

which survives in the unoriented theory. Since C_0 is odd under Ω and even under $\widehat{\Omega}$, we conclude that the same holds for the tachyon, namely

$$\begin{aligned}\Omega|(n,m);0;\Sigma_1\rangle &= -|(n,m);0;\Sigma_1\rangle \\ \widehat{\Omega}|(n,m);0;\Sigma_1\rangle &= |(n,m);0;\Sigma_1\rangle\,.\end{aligned} \tag{4.7}$$

[11] Our convention for η differs from that of [16] by a sign. Compare for example their equation (5.9) with our equation (A.1).

Therefore the tachyon is in the (anti)symmetric representation of the Sp(SO) gauge group in the Ω model, and the (anti)symmetric representation of the SO(Sp) gauge group in the $\widehat{\Omega}$ model. In addition there are scalar and fermion fields in the vector representation coming from the 0-1 strings. What is not known at this stage is which 0-brane gauge group goes with which 1-brane CP group. To determine this, we will derive the relation between the disk and RP^2 tadpoles from the 0-1 cylinder and 0-0 Möbius amplitudes, and compare with the relation obtained in section 3.

Let us start with the 0-1 cylinder amplitude. As there are two kinds of D1-brane, $D1_\pm$ in the Ω model and $\widetilde{D1}_\pm$ in the $\widehat{\Omega}$ model, there are two amplitudes to consider. The $\widetilde{D0}_-$-$D1_-$ (or $\widetilde{D1}_-$) strings are unprojected NS strings, and the $\widetilde{D0}_-$-$D1_+$ (or $\widetilde{D1}_+$) strings are unprojected R strings. The NS and R amplitudes are actually the same in two-dimensions, and given by[12]

$$\mathcal{A}_{C,\tilde{0}-1} = \frac{1}{2}n_0 n_1 \int_0^\infty \frac{dt}{2t}\int \frac{dp_0}{2\pi} e^{-\pi t p_0^2} e^{-\frac{\pi t}{4}\nu^2} = \frac{n_0 n_1}{8\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}} e^{-\frac{\pi^2}{4s}\nu^2} \tag{4.8}$$

$$\mathcal{A}_{C,\tilde{0}-\tilde{1}} = 2\mathcal{A}_{C,0-1}\,. \tag{4.9}$$

The factor of 2 for the $\widetilde{D0}_-$-$\widetilde{D1}_\eta$ amplitude is due to the fact that $\widetilde{D1}_\eta$ is a $D1_\eta\overline{D1}_\eta$ combination.

Now consider the $\widetilde{D0}_-$- $\widetilde{D0}_-$ Möbius amplitude. In the Liouville part, we need to sum over the states in the Verma module of $V_{n,m}$, and subtract the sum over the states in the null submodule. The Liouville Möbius partition function is given by

$$\begin{aligned} Z^L_{n,m}(t) = \mathrm{Tr}'_N\Big[&\langle (n,m),N;\mathbb{I}|\,P^+_{\mathrm{GSO}}\,\Omega\, e^{-2\pi t\left(L_0^L-\frac{\hat{c}_L}{16}\right)}\,|(n,m),N;\mathbb{I}\rangle \pm \\ &\pm \langle (n,m),N;\Sigma_1|\,P^-_{\mathrm{GSO}}\,\Omega\, e^{-2\pi t\left(L_0^L-\frac{\hat{c}_L}{16}\right)}\,|(n,m),N;\Sigma_1\rangle\Big]\,, \end{aligned} \tag{4.10}$$

where the upper and lower signs correspond to the Ω and $\widehat{\Omega}$ cases, respectively, as follows from (4.7). The Liouville hamiltonian is given by (with $\hat{c}_L = 1+2Q^2$)

$$L_0^{(L)} - \frac{\hat{c}_L}{16} = -\frac{1}{8}(nb+m/b)^2 + N_B + N_F - \frac{1}{16}\,, \tag{4.11}$$

and the primed trace denotes the subtraction of the null submodule. There are two points to make about the subtraction. First, the null submodule gets a factor of i^{nm} relative to the full Verma module. This comes from the action of Ω on the superconformal modes, $\Omega G_r \Omega = (-1)^r G_r$, in the NS sector. Second, the GSO projection is reversed relative to the full Verma module if nm is odd. These points are elaborated upon in appendix B. For $(n,m)=(1,1)$ we then get

$$Z^L_{1,1}(t) = -i\left(e^{\frac{\pi t}{4}(b+1/b)^2} \pm e^{\frac{\pi t}{4}(b-1/b)^2}\right) Z_{\mathrm{GSO}+} \mp \left(e^{\frac{\pi t}{4}(b+1/b)^2} \mp e^{\frac{\pi t}{4}(b-1/b)^2}\right) Z_{\mathrm{GSO}-}\,, \tag{4.12}$$

[12]We have included an extra factor of 1/2 so that the results we will find below are consistent with those in section 3. We will do so in the type 0A case as well. However, we do not have a clear understanding of how this factor appears from the open string viewpoint. In any case, what we are after is the relative sign of the tadpoles.

where the upper signs hold for Ω and the lower signs for $\widehat{\Omega}$, and where we have defined

$$Z_{\mathrm{GSO}\pm} \equiv \frac{\vartheta_{00}(0,it+1/2)^{1/2} \mp \vartheta_{01}(0,it+1/2)^{1/2}}{2e^{-i\pi/16}\eta(it+1/2)^{3/2}}\,. \tag{4.13}$$

Including $\hat{c}=1$ matter and ghosts (which cancel the oscillator contributions), and CP factors, and taking $b\to 1$, the Möbius amplitudes become

$$\mathcal{A}_{M,\tilde{0}-\tilde{0}} = \pm\frac{n_0}{4\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}}\left[-i\left(e^{\frac{\pi^2}{4s}}+1\right)\frac{1}{2}(1-1) - \left(e^{\frac{\pi^2}{4s}}-1\right)\frac{1}{2}(1+1)\right] \tag{4.14}$$

$$\mathcal{A}_{\widehat{M},\tilde{0}-\tilde{0}} = \pm\frac{n_0}{4\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}}\left[-i\left(e^{\frac{\pi^2}{4s}}-1\right)\frac{1}{2}(1-1) + \left(e^{\frac{\pi^2}{4s}}+1\right)\frac{1}{2}(1+1)\right], \tag{4.15}$$

for the Ω and $\widehat{\Omega}$ cases, respectively. The first of the two terms in the bracket comes from GSO-even states only, and vanishes since there are no propagating GSO-even states. We keep it with the factor $(1-1)$ to remind ourselves that the 1 comes from tracing over NS states with a $1\cdot\Omega$, and the -1 comes from tracing over NS states with a $(-1)^f\cdot\Omega$. The relative minus sign is due to the fact that the ground state of the NS sector is odd under $(-1)^f$.

Comparing now (4.9) with (4.15) in the $s\to\infty$ limit for the $\widehat{\Omega}$ case, we read off

$$\langle T\rangle_{\widehat{O1}} = \pm 2\langle T\rangle_{\widetilde{D1}_\eta}\,, \tag{4.16}$$

in agreement with what was found in (3.28). It follows that the D0-brane gauge group is $\mathrm{SO}(n_0)$ for $\widehat{\Omega}_+$ and $\mathrm{Sp}(n_0/2)$ for $\widehat{\Omega}_-$.

From the large s limit of the Ω amplitude we see that

$$\langle T\rangle_{O1} = 0\,. \tag{4.17}$$

This is consistent with the result in the previous section. However, in order to fix the D0-brane gauge group we need to compare (4.14) with (3.20) more carefully. In particular, the would-be tadpole corresponds to the leading term

$$\pm\frac{n_0}{4\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}}\left[-i(1-1)\right]. \tag{4.18}$$

As stated above, the 1 comes from an NS trace with $1\cdot\Omega$, and the -1 comes from an NS trace with $(-1)^f\cdot\Omega$. By channel duality, these correspond to the leading contributions of the tree amplitudes $\langle B,-|\Delta|C,+\rangle$ and $\langle B,-|\Delta|C,-\rangle$, respectively. The same holds for the 1 and -1 terms in (3.20). Given that the normalization of the D1-brane is fixed by the cylinder amplitude (4.8), we conclude that the gauge group assignment is again $\mathrm{SO}(n_0)$ for Ω_+, and $\mathrm{Sp}(n_0/2)$ for Ω_-.

4.2.2 The 0A models

Since we have fixed $\mu_0>0$, type 0A contains only a $D0_-$-brane. Recalling that C_1^- is odd under Ω and even under $\widehat{\Omega}$, it follows that the $D0_-$-brane is invariant under $\widehat{\Omega}$, and

JHEP01(2004)043

is mapped to the $\overline{D0}_-$-brane under Ω. We denote the invariant combination in the second case by $\widetilde{D0}_-$. The analysis of the gauge group and matter representation parallels the one for the 1-branes in type 0B (except that the roles of Ω and $\widehat{\Omega}$ are reversed). In the $\widehat{\Omega}$ theory the gauge group is either $SO(n_0) \times SO(\bar{n}_0)$ or $Sp(n_0/2) \times Sp(\bar{n}_0/2)$, the tachyon is in the bi-fundamental representation, and there are additional scalars and fermions in the vector representation. In the Ω theory the gauge group is $U(n_0)$ (for both $\Omega_\pm$), and the tachyon is either in the symmetric or antisymmetric representation. Here too there are additional scalars and fermions from the 0-1 strings. We will now compute the 0-1 cylinder and 0-0 Möbius amplitudes in order to fix which gauge group goes with which of $\widehat{\Omega}_\pm$, and which tachyon representation goes with which of $\Omega_\pm$.

The 0-1 cylinder amplitude is exactly the same as in the type 0B case, namely

$$\mathcal{A}_{C,0-\widetilde{1}} = n_0 n_1 \frac{1}{2}\int_0^\infty \frac{dt}{2t}\int \frac{dp_0}{2\pi} e^{-\pi t p_0^2} e^{-\frac{\pi t}{4}\nu^2} = \frac{n_0 n_1}{8\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}} e^{-\frac{\pi^2}{4s}\nu^2} \tag{4.19}$$

$$\mathcal{A}_{C,\widetilde{0}-\widetilde{1}} = 2\mathcal{A}_{C,0-\widetilde{1}}\,, \tag{4.20}$$

for the $\widehat{\Omega}$ and Ω model, respectively. Note that we included an extra factor of 1/2 as we remarked in footnote 12.

For the Möbius amplitude we again consider first the Liouville Möbius partition function for the general (n, m) Dirichlet state. In the $\widehat{\Omega}$ model it is given by

$$Z^{L,\widehat{\Omega}}_{n,m}(t) = \mathrm{Tr}'_N \,\langle (n,m), N|\, P^+_{\mathrm{GSO}}\,\Omega\, e^{-2\pi t\left(L_0^{(L)} - \frac{\hat{c}_L}{16}\right)}\,|(n,m), N\rangle\,, \tag{4.21}$$

where the primed trace denotes the subtraction of the null submodule. In the Ω model the $\widetilde{D0}_-$-$\widetilde{D0}_-$ strings have four CP sectors, just as in (3.21). The action of GSO and Ω on the CP factors is exactly the same as the action of GSO and $\widehat{\Omega}$ for the $\widetilde{D1}_\pm$-brane in type 0B. Therefore the Liouville partition function, after summing over the CP factors, yields

$$Z^{L,\Omega}_{n,m}(t) = \pm 2\,\mathrm{Tr}'_N \,\langle (n,m), N|\, P^-_{\mathrm{GSO}}\,\Omega\, e^{-2\pi t\left(L_0^{(L)} - \frac{\hat{c}_L}{16}\right)}\,|(n,m), N\rangle\,, \tag{4.22}$$

where the choice of sign corresponds to $\Omega_\pm$, defined in analogy with (3.26). The computation proceeds in parallel to the type 0B case. For $(n, m) = (1, 1)$ one finds

$$Z^{L,\widehat{\Omega}}_{1,1}(t) = -ie^{\frac{\pi t}{4}(b+1/b)^2} Z_{\mathrm{GSO}+} + e^{\frac{\pi t}{4}(b-1/b)^2} Z_{\mathrm{GSO}-}\,, \tag{4.23}$$

$$Z^{L,\Omega}_{1,1}(t) = \pm 2\left(e^{\frac{\pi t}{4}(b+1/b)^2} Z_{\mathrm{GSO}-} + ie^{\frac{\pi t}{4}(b-1/b)^2} Z_{\mathrm{GSO}+}\right). \tag{4.24}$$

Including the $\hat{c} = 1$ matter and ghosts, and taking $b \to 1$, the Möbius amplitudes become

$$\mathcal{A}_{\widehat{M},0-0} = \pm\frac{n_0}{4\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}} \tag{4.25}$$

$$\mathcal{A}_{M,\widetilde{0}-\widetilde{0}} = \pm\frac{2n_0}{4\pi^{3/2}}\int_0^\infty \frac{ds}{s^{1/2}} e^{\frac{\pi^2}{4s}}\,, \tag{4.26}$$

for the $\widehat{\Omega}$ and Ω models, respectively. In the fomer case the sign determines the gauge group as $SO(n_0)$ or $Sp(n_0/2)$, whereas in the latter case the gauge group is $U(n_0)$, and

JHEP01(2004)043

the sign determines the tachyon representation. Comparing (4.25) with (4.19), and (4.26) with (4.20) in the $s \to \infty$ limit, one can read off that

$$\langle T \rangle_{\widehat{O1}} = \pm 2 \langle T \rangle_{\widetilde{D1}_\eta} \tag{4.27}$$

$$\langle T \rangle_{O1} = \pm 2 \langle T \rangle_{\widetilde{D1}_\eta} \,. \tag{4.28}$$

Now comparing these with (3.39) and (3.38), we conclude that in $0\mathrm{A}/\widehat{\Omega}$ the D0-brane gauge group is $\mathrm{SO}(n_0) \times \mathrm{SO}(\bar{n}_0)$ for $\widehat{\Omega}_+$ and $\mathrm{Sp}(n_0/2) \times \mathrm{Sp}(\bar{n}_0/2)$ for $\widehat{\Omega}_-$, and that in $0\mathrm{A}/\Omega$ the tachyon is symmetric for Ω_+ and antisymmetric for Ω_-. This is shown, together with the fundamental scalars and fermions, in table 3.

5. Discussion

We have classified the possible unoriented string theories in two dimensions, and constructed their (conjectured) dual matrix models. The tadpole-free theories are listed in table 1. Unlike RR tadpoles, the massless tachyon tadpole is not inconsistent, but rather is a source of quantum corrections to the classical (tachyon) background [46]. Thus the tadpole-non-free theories are as sensible as the tadpole-free ones, unless the quantum corrections lead to a runaway tachyon potential. Whereas in critical string theory it is not easy to implement the Fischler-Susskind mechanism beyond a small number of loops, we expect that in two dimensions the exact quantum corrected background should be built-in to the dual matrix models. It would be interesting to systematically extract the quantum shifts of the classical background from a matrix model computation.

In the (bosonic and type 0B) oriented case without D1-branes the dual matrix models are equivalent to free fermions in an inverted harmonic oscillator potential. The background is represented by the Fermi surface. It is therefore conceivable that matrix models for tadpole-free theories in general can be described by non-interacting fermions. It is also possible that matrix models for theories with non-vanishing tadpoles may have a description in terms of interacting fermions [47, 48, 49, 50]. It would be very interesting to study the unoriented matrix models in more detail.

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A. Channel duality

The cylinder, Möbius strip, and Klein bottle amplitudes can be expressed either as one-loop (open or closed) vacuum amplitudes, or as tree level amplitudes corresponding to the propagation of closed strings between boundary states and/or crosscap states. We refer to the relation between the loop-channel and tree-channel descriptions as *channel duality.* In this appendix we shall review the channel duality map for the world-sheet fermions.

JHEP01(2004)043

The results are summarized in tables (4), (5) and (6), where we translate the different spin structures of the wordlsheet fermions into the tree and loop channels for the cylinder, Möbius strip and Klein bottle. The translation tables are equally valid for critical and non-critical superstrings, as they only depend on the topology of the given world-sheet.

A.1 Cylinder

Let us begin with the cylinder. We have to impose two boundary conditions and a periodicity condition on the world-sheet fermions:

$$\begin{array}{lrcl} \text{Boundary I} & \tilde\psi(0,\sigma^2) &=& \eta_1\psi(0,\sigma^2) \\ \text{Boundary II} & \tilde\psi(s,\sigma^2) &=& \eta_2\psi(s,\sigma^2) \\ \text{Periodicity} & \psi(\sigma^1,2\pi) &=& \eta_3\psi(\sigma^1,0) \\ & \tilde\psi(\sigma^1,2\pi) &=& \eta_4\tilde\psi(\sigma^1,0), \end{array} \tag{A.1}$$

where $\eta_i = \pm 1$. The four phases are not independent however. Combining one of the boundary conditions with the periodicity condition applied to that boundary we infer that $\eta_3 = \eta_4 \equiv \eta'$. In the tree channel σ^1 plays the role of time and σ^2 plays the role of space. From this point of view the above conditions define boundary states with spin structures $|B_I, \eta_1\rangle_{\eta'}$ and $|B_{II}, \eta_2\rangle_{\eta'}$, where $\eta' = \pm 1$ refers to the R-R and NS-NS sector, respectively. Actually, since we can multiply the left-movers relative to the right-movers by a phase, the amplitude only depends on the product $\eta_1\eta_2 \equiv \eta$, so there are only four independent spin structures parameterized by η and η'. In the loop channel the roles of σ^1 and σ^2 are interchanged, so we now find that $\eta = \pm 1$ corresponds to the open string R and NS sector, respectively, and $\eta' = \pm 1$ corresponds to the insertion of $(-1)^f$ and 1 in the open string trace, respectively.

(η', η)	tree channel	loop channel
$(+,+)$	$\langle B,\pm\vert\Delta\vert B,\pm\rangle_{\mathrm{RR}}$	NS $(-1)^f$
$(+,-)$	$\langle B,\pm\vert\Delta\vert B,\mp\rangle_{\mathrm{RR}}$	R $(-1)^f$
$(-,+)$	$\langle B,\pm\vert\Delta\vert B,\pm\rangle_{\mathrm{NSNS}}$	NS 1
$(-,-)$	$\langle B,\pm\vert\Delta\vert B,\mp\rangle_{\mathrm{NSNS}}$	R 1

Table 4: Cylinder channel duality.

A.2 Möbius strip

In the case of the Möbius strip we have to impose one boundary condition, one crosscap condition and a periodicity condition:

$$\begin{array}{lrcl} \text{Boundary} & \tilde\psi(0,\sigma^2) &=& \eta_1\psi(0,\sigma^2) \\ \text{Crosscap} & \psi(s,\sigma^2) &=& \eta_2\tilde\psi(s,\sigma^2+\pi) \\ & \tilde\psi(s,\sigma^2) &=& \eta_3\psi(s,\sigma^2+\pi) \\ \text{Periodicity} & \psi(\sigma^1,2\pi) &=& \eta_4\psi(\sigma^1,0) \\ & \tilde\psi(\sigma^1,2\pi) &=& \eta_5\tilde\psi(\sigma^1,0)\,. \end{array} \tag{A.2}$$

As in the case of the cylinder, there are only four independent spin structures, which one can take to be parameterized by $\eta = \eta_1\eta_2$ and $\eta' = \eta_4$; multiplication of the left-movers by an overall phase fixes, say, $\eta_2 = +1$, the boundary condition then forces $\eta_5 = \eta'$, and the crosscap condition (applied twice) requires $\eta_3 = \eta'$. In the tree channel we then have a boundary state $|B,\eta_1\rangle'_\eta$ and a crosscap state $|C,\eta_2\rangle'_\eta$, where, as before, $\eta' = \pm 1$ corresponds to the R-R and NS-NS sector, respectively. In the loop channel the Möbius strip corresponds to a twisted identification of two opposite boundaries of a rectangle. To get this we first double the range of σ^1 to $[0, 2s]$ by copying the (σ^1, σ^2) plane and reflecting the copy about the σ^1 axis, and then halve the range of σ^2 to $[0, \pi]$. In the new domain we define

$$\begin{aligned} \Psi(\sigma^1,\sigma^2) &\equiv \begin{cases} \psi(\sigma^1,\sigma^2) & \sigma^1 < s \\ \tilde\psi(2s-\sigma^1,\sigma^2+\pi) & \sigma^1 > s \end{cases} \\ \tilde\Psi(\sigma^1,\sigma^2) &\equiv \begin{cases} \tilde\psi(\sigma^1,\sigma^2) & \sigma^1 < s \\ \eta'\psi(2s-\sigma^1,\sigma^2+\pi) & \sigma^1 > s\,. \end{cases} \end{aligned} \tag{A.3}$$

The conditions (A.2) then imply the boundary condition

$$\begin{aligned} \Psi(0,\sigma^2) &= \eta\tilde\Psi(0,\sigma^2) \\ \Psi(2s,\sigma^2) &= \eta\eta'\tilde\Psi(2s,\sigma^2) \end{aligned}$$

and the twisted periodicity condition

$$\begin{aligned} \Psi(\sigma^1,0) &= \eta'\tilde\Psi(2s-\sigma^1,\pi) \\ \tilde\Psi(\sigma^1,0) &= \Psi(2s-\sigma^1,\pi)\,. \end{aligned}$$

By redefining the left-mover by a phase η we see that $\eta' = \pm 1$ corresponds to the R and NS open string sector, respectively, and that $\eta = \pm 1$ corresponds to the two possible insertions $(-1)^f \cdot \Omega$ and $1 \cdot \Omega$. It isn't immediately clear which insertion corresponds to which sign of η, but an explicit calculation of the amplitude shows that $\eta = +1$ corresponds to $(-1)^f \cdot \Omega$, and $\eta = -1$ to $1 \cdot \Omega$ (see for example [51], where this was done for the critical superstring).

(η',η)	tree channel	loop channel
$(+,+)$	$\langle B,\pm\vert\Delta\vert C,\pm\rangle_{\mathrm{RR}}$	R $(-1)^f\cdot\Omega$
$(+,-)$	$\langle B,\pm\vert\Delta\vert C,\mp\rangle_{\mathrm{RR}}$	R $1\cdot\Omega$
$(-,+)$	$\langle B,\pm\vert\Delta\vert C,\pm\rangle_{\mathrm{NSNS}}$	NS $(-1)^f\cdot\Omega$
$(-,-)$	$\langle B,\pm\vert\Delta\vert C,\mp\rangle_{\mathrm{NSNS}}$	NS $1\cdot\Omega$

Table 5: Möbius strip channel duality.

A.3 Klein bottle

The Klein bottle requires two crosscap conditions and a periodicity condition:

$$\begin{array}{lrl} \text{Crosscap I} & \psi(0,\sigma^2) &= \eta_1\tilde\psi(0,\sigma^2+\pi) \\ & \tilde\psi(0,\sigma^2) &= \eta_2\psi(0,\sigma^2+\pi) \\ \text{Crosscap II} & \psi(s,\sigma^2) &= \eta_3\tilde\psi(s,\sigma^2+\pi) \\ & \tilde\psi(s,\sigma^2) &= \eta_4\psi(s,\sigma^2+\pi) \\ \text{Periodicity} & \psi(\sigma^1,2\pi) &= \eta_5\psi(\sigma^1,0) \\ & \tilde\psi(\sigma^1,2\pi) &= \eta_6\tilde\psi(\sigma^1,0)\,. \end{array} \tag{A.4}$$

Once again, there are only four independent spin structures, which we take to be parameterized by $\eta = \eta_1\eta_3$ and $\eta' = \eta_5$. We can use the freedom to redefine the left-mover by an

(η',η)	tree channel	loop channel
$(+,+)$	$\langle C,\pm\vert\Delta\vert C,\pm\rangle_{\mathrm{RR}}$	R-R $((-1)^f+(-1)^{\tilde f})\cdot\Omega$
$(+,-)$	$\langle C,\pm\vert\Delta\vert C,\mp\rangle_{\mathrm{RR}}$	NS-NS $((-1)^f+(-1)^{\tilde f})\cdot\Omega$
$(-,+)$	$\langle C,\pm\vert\Delta\vert C,\pm\rangle_{\mathrm{NSNS}}$	R-R $(1+(-1)^{f+\tilde f})\cdot\Omega$
$(-,-)$	$\langle C,\pm\vert\Delta\vert C,\mp\rangle_{\mathrm{NSNS}}$	NS-NS $(1+(-1)^{f+\tilde f})\cdot\Omega$

Table 6: Klein bottle channel duality.

overall phase to fix $\eta_3=+1$. Then the constraints from applying the two crosscap conditions twice give $\eta_2=\eta\eta'$ and $\eta_4=\eta_6=\eta'$. The above conditions then define two crosscap states $|C_I,\eta_1\rangle'_\eta$ and $|C_{II},\eta_3\rangle'_\eta$ in the tree channel. For the loop channel we again have to work in the domain $\sigma^1\in[0,2s]$ and $\sigma^2\in[0,\pi]$. The fermions in the new domain are then defined precisely as in (A.3). We now get a periodicity condition in the σ^1 direction,

$$\begin{aligned}\Psi(0,\sigma^2) &= \eta\Psi(2s,\sigma^2)\\ \tilde\Psi(0,\sigma^2) &= \eta\tilde\Psi(2s,\sigma^2)\,,\end{aligned}\tag{A.5}$$

and a twisted periodicity condition in the σ^2 direction,

$$\begin{aligned}\Psi(\sigma^1,0) &= \eta'\tilde\Psi(2s-\sigma^1,\pi)\\ \tilde\Psi(\sigma^1,0) &= \Psi(2s-\sigma^1,\pi)\,.\end{aligned}\tag{A.6}$$

Therefore $\eta=\pm1$ corresponds to the (closed string) R-R and NS-NS sector, respectively, and $\eta'=\pm1$ corresponds to the insertion of $((-1)^f+(-1)^{\tilde f})\cdot\Omega$ and $(1+(-1)^{f+\tilde f})\cdot\Omega$ in the closed string trace, respectively.[13]

B. Some relevant calculations

In this appendix, we show some details of the computations.

B.1 The GSO projected character of degenerate representation on $\mathbb{RP}^2$

In section 4, we computed the character of degenerate representation on $\mathbb{RP}^2$. Here we will elaborate the details of the calculation in the type 0B case. The generalization to the type 0A case is straightforward. Let us define the GSO projected character on $\mathbb{RP}^2$ by

$$\begin{aligned}\chi^{\mathrm{GSO}\pm}_{\mathbb{I}}(\Delta) &\equiv \mathrm{Tr}_{\mathbb{I}}\left(P^{\pm}_{\mathrm{GSO}}\,\Omega\,q^{\Delta+N_B+N_F}\right),\\ \chi^{\mathrm{GSO}\pm}_{\Sigma_1}(\Delta) &\equiv \mathrm{Tr}_{\Sigma_1}\left(P^{\pm}_{\mathrm{GSO}}\,\Omega\,q^{\Delta+N_B+N_F}\right).\end{aligned}\tag{B.1}$$

The trace is over the states in the $\mathbb{I}$ sector for $\chi_{\mathbb{I}}$, and those in the Σ_1 sector for χ_{Σ_1}, where two CP sectors are defined in section 4. Recall the difference of the Ω-action on the

[13]The precise correspondence is again verified by an explicit calculation [51]. It may seem odd that the Klein bottle has only four spin structures rather than eight (like the torus), corresponding in the loop channel to the choice of R-R or NS-NS sector, and to the separate insertions Ω, $(-1)^f\Omega$, $(-1)^{\tilde f}\Omega$ or $(-1)^{f+\tilde f}\Omega$. This is because Ω relates the left-moving state to the right-moving state, and in particular identifies the left and right-moving spin structures.

ground state (thus the highest weight state) in each sector. In order to understand how to subtract the null submodule, let us expand the characters and examine the low order terms (we consider the $(1,1)$ degenerate module, but the generalization to the (n,m) case is straightforward):

$$\begin{aligned}\chi_{\mathbb{I}}^{\mathrm{GSO}\pm}(\Delta) &= -\frac{1}{2}i\left[(1-iq^{1/2}+\cdots)\pm(-1-iq^{1/2}+\cdots)\right]q^{\Delta}\,,\\ \chi_{\Sigma_1}^{\mathrm{GSO}\pm}(\Delta) &= \frac{1}{2}\left[(1-iq^{1/2}+\cdots)\pm(-1-iq^{1/2}+\cdots)\right]q^{\Delta}\,.\end{aligned} \tag{B.2}$$

The terms of $q^{1/2}$ corresponds to the null states in the case $(n,m)=(1,1)$. We would like to compare these with the character of the null module

$$\begin{aligned}\chi_{\mathbb{I}}^{\mathrm{GSO}\pm}\left(\Delta+\frac{1}{2}\right) &= -\frac{1}{2}iq^{1/2}\left[(1-iq^{1/2}+\cdots)\pm(-1-iq^{1/2}+\cdots)\right]q^{\Delta}\,,\\ \chi_{\Sigma_1}^{\mathrm{GSO}\pm}\left(\Delta+\frac{1}{2}\right) &= \frac{1}{2}q^{1/2}\left[(1-iq^{1/2}+\cdots)\pm(-1-iq^{1/2}+\cdots)\right]q^{\Delta}\,.\end{aligned} \tag{B.3}$$

Then we see that the subtraction should be taken as

$$\chi_{\mathbb{I},\Sigma_1}^{\mathrm{GSO}\pm}(\Delta)+i\chi_{\mathbb{I},\Sigma_1}^{\mathrm{GSO}\mp}\left(\Delta+\frac{1}{2}\right). \tag{B.4}$$

Note the following two points; We need a factor of i, and the GSO projection must be reversed in the subtraction. For the (n,m) case, the factor of i be replaced by i^{nm}, and the GSO projection is reversed for nm odd and the same for nm even.

Then the Liouville partition function as defined in (4.10) yields

$$\begin{aligned}Z^L_{n,m}(t) = -i&\left\{e^{\frac{\pi t}{4}(nb+m/b)2}Z_{\mathrm{GSO},+}+i^{nm}e^{\frac{\pi t}{4}(nb-m/b)2}Z^{n,m}_{\mathrm{GSO},-}\right\}\mp\\ &\mp\left\{e^{\frac{\pi t}{4}(nb+m/b)2}Z_{\mathrm{GSO},-}+i^{nm}e^{\frac{\pi t}{4}(nb-m/b)2}Z^{n,m}_{\mathrm{GSO},+}\right\}.\end{aligned} \tag{B.5}$$

where $Z_{\mathrm{GSO},\pm}$ are defined in eq. (4.13) and we have defined

$$Z^{n,m}_{\mathrm{GSO},\pm}\equiv\frac{\vartheta_{00}(0,it+1/2)^{1/2}\mp(-1)^{nm}\vartheta_{01}(0,it+1/2)^{1/2}}{2e^{-i\pi/16}\eta(it+1/2)^{3/2}}\,. \tag{B.6}$$

B.2 Boundary and cross-cap states

The boundary states in the super Liouville theory were constructed in [44, 45], by solving the boundary bootstrap equations obtained by making use of the degenerate conformal field and imposing the Cardy condition, generalizing the work of [33, 10]. In this appendix, we will not perform the computation of one-point functions. Instead we use the result obtained in [44, 45] as an input for constructing, in particular, the cross-cap states.[14] We only consider the NS-NS sector, which is relevant for our analysis.

[14]The crosscap state for the two-dimensional bosonic string was constructed in [35].

The building block of the boundary and cross-cap states is the Ishibashi states that have the following properties,

$$ {}_{\rm NSNS}\langle B;P',\pm|\Delta|B;P,\pm\rangle_{\rm NSNS} = \delta(P'+P)\frac{\vartheta_{00}(0,is/\pi)^{1/2}}{\eta(is/\pi)^{3/2}}\,, \tag{B.7} $$

$$ {}_{\rm NSNS}\langle B;P',\pm|\Delta|B;P,\mp\rangle_{\rm NSNS} = \delta(P'+P)\frac{\vartheta_{01}(0,is/\pi)^{1/2}}{\eta(is/\pi)^{3/2}}\,, \tag{B.8} $$

$$ {}_{\rm NSNS}\langle B;P',\pm|\Delta|C;P,\pm\rangle_{\rm NSNS} = \delta(P'+P)\frac{\vartheta_{00}(0,is/\pi+1/2)^{1/2}}{e^{-i\pi/16}\eta(is/\pi+1/2)^{3/2}}\,, \tag{B.9} $$

$$ {}_{\rm NSNS}\langle B;P',\pm|\Delta|C;P,\mp\rangle_{\rm NSNS} = \delta(P'+P)\frac{\vartheta_{01}(0,is/\pi+1/2)^{1/2}}{e^{-i\pi/16}\eta(is/\pi+1/2)^{3/2}}\,, \tag{B.10} $$

$$ {}_{\rm NSNS}\langle C;P',\pm|\Delta|C;P,\pm\rangle_{\rm NSNS} = \delta(P'+P)\frac{\vartheta_{00}(0,is/\pi)^{1/2}}{\eta(is/\pi)^{3/2}}\,, \tag{B.11} $$

$$ {}_{\rm NSNS}\langle C;P',\pm|\Delta|C;P,\mp\rangle_{\rm NSNS} = \delta(P'+P)\frac{\vartheta_{01}(0,is/\pi)^{1/2}}{\eta(is/\pi)^{3/2}}\,, \tag{B.12} $$

where we have defined $\Delta \equiv e^{-s(N+\widetilde{N}-1/8)}$. $N = N_B + N_F$ is the sum of bosonic and fermionic parts of the Virasoro level operator and likewise for $\widetilde{N}$.

The NS-NS boundary state for the $(1,1)$ Dirichlet brane is

$$ |B_{1,1},-\rangle_{\rm NSNS} = \int_0^\infty dP\,\Psi^{\rm NS}_{1,1}(P)|B;P,-\rangle_{\rm NSNS}\,, \tag{B.13} $$

where the boundary state wave-function $\Psi^{\rm NS}_{1,1}(P)$ is given by [44, 45]

$$ \Psi^{\rm NS}_{1,1}(P) = 4\left(\pi\mu\gamma\left(\frac{bQ}{2}\right)\right)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-2i\pi P}\sinh(\pi Pb)\sinh\left(\frac{\pi P}{b}\right). \tag{B.14} $$

The Liouville partition function for the 0-1 open string is given by

$$ \begin{aligned} Z^{\rm annulus}_{\alpha=Q/2+i\nu/2} &\equiv \mu_1\,{\rm Tr}_{\rm NS}\ e^{-2\pi t\left(L_0^{(L)}-\frac{\hat{c}}{16}\right)} = \mu_1 e^{-\frac{\pi t}{4}\nu 2}\frac{\vartheta_{00}(0,it)^{1/2}}{\eta(it)^{3/2}} \\ &= \mu_1\left(\frac{\pi}{s}\right)^{1/2} e^{-\frac{\pi 2}{4s}\nu 2}\frac{\vartheta_{00}(0,is/\pi)^{1/2}}{\eta(is/\pi)^{3/2}} \\ &= 2\mu_1\int_0^\infty dP e^{-sP2}\frac{\vartheta_{00}(0,is/\pi)^{1/2}}{\eta(is/\pi)^{3/2}}\cos(\pi\nu P)\,, \end{aligned} \tag{B.15} $$

where $\mu_1 = 2$ for $\widetilde{D0}-\widetilde{D1}$ in both type 0A and 0B, and $\mu_1 = 1$ for $D0-\widetilde{D1}$ in type 0A, and $\widetilde{D0}-D1$ in type 0B. This determines the NS-NS boundary state for the Neumann brane to be

$$ |B_\nu,-\rangle_{\rm NSNS} = \int_0^\infty dP\,\Psi^{\rm NS}_\nu(P)|B;P,-\rangle_{\rm NSNS}\,, \tag{B.16} $$

where the boundary state wave-function $\Psi^{\rm NS}_\nu(P)$ is given by [44, 45]

$$ \Psi^{\rm NS}_\nu(P) = \mu_1\left(\pi\mu\gamma\left(\frac{bQ}{2}\right)\right)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-i\pi P}\cos(\pi\nu P)\,. \tag{B.17} $$

The Liouville partition function (B.5) on $\mathbb{RP}^2$ for type 0B takes, in the tree channel, the form

$$Z^L_{1,1} = \left(\frac{\pi}{2s}\right)^{1/2}\left[-i\left(e^{\frac{\pi^2}{16s}(b+1/b)^2} \pm e^{\frac{\pi^2}{16s}(b-1/b)^2}\right)Z^{BC-} \mp \left(e^{\frac{\pi^2}{16s}(b+1/b)^2} \mp e^{\frac{\pi^2}{16s}(b-1/b)^2}\right)Z^{BC+}\right], \tag{B.18}$$

where $s = \pi/4t$ and we have defined

$$Z^{BC\pm} \equiv \frac{\vartheta_{01}(0, is/\pi + 1/2)^{1/2} \pm e^{-\pi i/4}\vartheta_{00}(0, is/\pi + 1/2)^{1/2}}{2e^{-3i\pi/16}\eta(is/\pi + 1/2)^{3/2}}\,. \tag{B.19}$$

It can be further rewritten as

$$\begin{aligned} Z^L_{1,1} = \sqrt{2}\int_0^\infty dP e^{-sP2}\Bigg[&-i\left(\cosh\left(\frac{\pi(b+1/b)P}{2}\right) \pm \cosh\left(\frac{\pi(b-1/b)P}{2}\right)\right)Z^{BC-} \mp \\ &\mp \left(\cosh\left(\frac{\pi(b+1/b)P}{2}\right) \mp \cosh\left(\frac{\pi(b-1/b)P}{2}\right)\right)Z^{BC+}\Bigg]. \end{aligned} \tag{B.20}$$

Then the NS-NS cross-cap state can be read off as (up to irrelevant phase factors which will be cancelled upon the inclusion of the matter and ghost parts)

$$\begin{aligned} |C\rangle_{\rm NSNS} = \frac{1}{2}\int_0^\infty dP\Big[&\Psi^{\rm NS}_{C-}(P)\left(|C;P,+\rangle_{\rm NSNS} - |C;P,-\rangle_{\rm NSNS}\right) + \\ &+ \Psi^{\rm NS}_{C+}(P)\left(|C;P,+\rangle_{\rm NSNS} + |C;P,-\rangle_{\rm NSNS}\right)\Big]. \end{aligned} \tag{B.21}$$

The cross-cap state wave-function $\Psi^{\rm NS}_{C\pm}(P)$ is found to be

$$\begin{aligned} \Psi^{\rm NS}_{C-}(P) &= -2\sqrt{2}i\left(\pi\mu\gamma\left(\frac{bQ}{2}\right)\right)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-2i\pi P}\sinh\left(\frac{\pi Pb}{2}\right)\sinh\left(\frac{\pi P}{2b}\right), \\ \Psi^{\rm NS}_{C+}(P) &= 2\sqrt{2}\left(\pi\mu\gamma\left(\frac{bQ}{2}\right)\right)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-2i\pi P}\cosh\left(\frac{\pi Pb}{2}\right)\cosh\left(\frac{\pi P}{2b}\right), \end{aligned}$$

for the upper sign ($0B/\hat{\Omega}$ model), and

$$\begin{aligned} \Psi^{\rm NS}_{C-}(P) &= -2\sqrt{2}i\left(\pi\mu\gamma\left(\frac{bQ}{2}\right)\right)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-2i\pi P}\cosh\left(\frac{\pi Pb}{2}\right)\cosh\left(\frac{\pi P}{2b}\right), \\ \Psi^{\rm NS}_{C+}(P) &= -2\sqrt{2}\left(\pi\mu\gamma\left(\frac{bQ}{2}\right)\right)^{-iP/b}\frac{\Gamma(1+iPb)\Gamma(1+iP/b)}{-2i\pi P)}\sinh\left(\frac{\pi Pb}{2}\right)\sinh\left(\frac{\pi P}{2b}\right), \end{aligned}$$

for the lower sign ($0B/\Omega$ model).

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